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ACTIVE METHODS OF MATHEMATICS EDUCATION

PETER VANKÚŠ¹, ANNA MATFIAKOVÁ

ABSTRACT. *In the paper we deal with the survey on active methods of mathematics education. The survey was done by the research method of interview with the structure of 10 items with more possible choices and 1 item with open answer. The participants of the interview had possibility to further clarify their responses verbally to make the answers more personal. The participants were 20 teachers of selected schools in Bratislava. In the paper we describe the survey, used items, and we analyze the answers. They give us insight to the active methods used by respondents in their mathematics lessons. Survey also provided us with participants' opinions about efficacy of active methods in various years of school education, various stages of the lesson and also sources of the active methods those are these teachers using. The results of the survey showed us that participating teachers are using all sort of active methods and they are aware of the importance of such methods for mathematics education. They tend to use these methods almost in all years of education and in various areas of mathematics. Respondents are also using many sources of active methods, even creating their own.*

KEY WORDS: *active methods, mathematics education, survey*

CLASSIFICATION: *D40, C70*

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Introduction

One of the most important tasks for the teacher is to motivate the students and encourage them to play an active role in education. The teacher has a number of options to fulfil this role. One of them is the use of active teaching methods.

In the literature, the notion of active teaching methods most often denote the teaching methods, those plan, organize and manage teaching in the way that educational goals are fulfilled mostly thanks to the own cognitive activities of students [1]. Many researches support the value of active learning [2], [3], [4]. From the previous study realized on the sample of 1248 teachers in Slovakia we also know, that about 80% of them see the importance of using innovations in their teaching, what in this study include also active methods [5]. There exist very useful sources of activities those belong to the active teaching methods available for mathematics teachers in Slovakia. We can mention for example publications containing various such activities proper for the use in mathematics education [6], [7], [8], [9].

The positive results of researches and available resources encourage using of active teaching methods by mathematics teachers in Slovakia. In this article we describe survey on actual teachers' practice of selected sample of teachers in this school subject.

The aim of our survey was to determine if selected mathematics teachers in Bratislava apply active teaching methods, which of them they use, in which conditions and also to study specific active methods they apply in their mathematics lessons. Because of the small sample we cannot make any general conclusions but we can get insight to the using of active teaching methods of selected teachers and also their best practices. This insight

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and examples of best practices can contribute with valuable ideas to educators developing materials for active learning and also to other mathematics teachers in practice.

Survey sample, administration and results

Survey respondents were teachers of mathematics at primary and secondary schools (mostly grammar schools) in Bratislava. We conducted interviews with 20 teachers of mathematics, of which 6 were men and 14 were women. Of them 4 taught only in lower secondary education, while 7 at lower secondary and upper secondary education (8 years grammar school). Only in upper secondary education worked 9 respondents. Most of the respondents (16) were experienced teacher with more than 10 years of teaching experience, 4 respondents have lower than 10 years of teaching experience.

The survey was carried out in the form of the interview with the given structure of items. Before realization of the survey we made testing interview to find out if the items are proper and clear. On the basis of this we made the final version of the interview with modified items worded more clearly and accurately. We have also extended the offer of possible answers for some items. Interview that we have created comprises of 10 closed questions with multiple choice answers and one open question. For all closed questions, except the first, each respondent was allowed to choose more than one from several offered options. The items of the interview are in the table 1.

In the beginning of the interview we obtained each respondent's data needed to characterize the research sample. Then we followed with items about the use of active teaching methods in mathematics. The interviews were recorded on a digital voice recorder. Consequently, we analyzed and quantified obtained data. We tried to interpret them in the light of the opinions and attitudes expressed by teachers verbally during interviews.

The following table shows the numbers of positive answers for the items of the interview.

Q1	Do you use active methods in mathematics education? (By active methods we mean teaching methods that somehow motivate students to be active in the lesson or they arouse their interest in mathematics.)	
a)	yes	19
b)	no	1
In the case, that answer for Q1 was yes:		
Q2	Which active methods you use in the education?	
a)	didactic games and competitions	15
b)	interesting lectures (with mathematics theme)	5
c)	discussion with mathematician or with expert	2
d)	using interesting and actual tasks	14
e)	real-life based tasks using students' experience	17
f)	historical notes	13
g)	using some ICT – software, voting machines, interactive tables, applets	15
h)	excursion	2
i)	quick quiz, warm-up exercise	11
Q3	Which form of the teaching you use when applying active methods?	
a)	frontal teaching	16
b)	individual teaching	7
c)	group teaching	17
Q4	In teaching of what mathematics areas you use active methods?	
a)	all	13
b)	numbers, variables, computations with numbers	3
c)	expressions, functions, tables, diagrams	3
d)	geometry and measurement	5
e)	combinatorics, probability and statistics	5
f)	logic, reasoning and proofs	1

Q5	In teaching of what mathematics areas you perceive your using of active methods most suitable and most effective?	
a)	all	4
b)	numbers, variables, computations with numbers	6
c)	expressions, functions, tables, diagrams	6
d)	geometry and measurement	8
e)	combinatorics, probability and statistics	6
f)	logic, reasoning and proofs	2
Q6	In what years of study you use active methods?	
a)	in all years of study (lower secondary and upper secondary)	11
b)	in first three years of lower secondary (respondent teaches at lower secondary)	2
c)	in last two years of lower secondary (respondent teaches at lower secondary)	1
d)	in first three years of upper secondary (respondent teaches at upper secondary)	5
e)	in last year of upper secondary (respondent teaches at upper secondary)	1
Q7	In what years of study you perceive active methods working the best?	
a)	in all years of study (lower secondary and upper secondary)	9
b)	in first three years of lower secondary (respondent teaches at lower secondary)	2
c)	in last two years of lower secondary (respondent teaches at lower secondary)	2
d)	in first three years of upper secondary (respondent teaches at upper secondary)	6
e)	in last year of upper secondary (respondent teaches at upper secondary)	2
Q8	In which phase of mathematics lesson you use mostly active methods?	
a)	about the same in each	11
b)	in motivational phase	6
c)	in exposition phase	4
d)	in exercising phase	2
e)	in repetition phase	2
f)	as a homework	0
Q9	In the activities those belong to active teaching methods you put attention that tasks are focused on:	
a)	work with information	6
b)	that the contexts of tasks are actual and based on reality	12
c)	that the context is connected with scientific world	1
d)	work with ICT	8
e)	development of pupils ability to communicate mathematics concepts and to use proper mathematics language	10
Q10	When planning active methods to include in your lesson you use:	
a)	existing collections of tasks and activities	11
b)	online sources of activities	14
c)	you create activities on your own	14
d)	you use activities from your colleagues, those worked well for them	8
e)	proper popular or scientific literature	8
Q11	State concrete examples of active methods those you use on your mathematics lessons and the mathematics areas where you use these activities.	
In the case, that answer for Q1 was <u>no</u>:		
N1	Why you do not use active teaching methods?	
a)	do not have time to prepare them	1
b)	I think that mathematics is enough interesting on its own	0
c)	my pupils are enough interested in mathematics without using these methods	1
N2	What activities you use in your mathematics lessons?	
a)	from the textbooks	1
b)	I use tasks from different sources	1
c)	I use real-life and practical tasks	1

Table 1: Items of the interview with numbers of positive answers

Discussion

In this part of the paper we will discuss the answers to the items of the interview.

Q1: All respondents, except one, use active teaching methods. Some respondents expressed the opinion that it is one of the most essential and important tasks for the teachers. Some respondents try to incorporate at each lesson activity that is motivational and makes students active and find this as important.

One respondent who stated not using active methods is the teacher of mathematically gifted students in mathematics orientated classes, and according his opinion for activation of these students is sufficient suitable choice of mathematical tasks and problems, with emphasis on the connection with life and real examples from practice.

Q2: We can say that some active methods are used by most teachers (see results in table 1). Interesting is fact that majority of the teachers said that they use as an active method also quick quizzes or warm-up exercises. Some respondents consider these methods as warm-up and preparation for mathematics lesson and their results are not included in the student assessment. Other teachers understand them as a means of extrinsic motivation, and these quizzes are part of student assessment. Here we can see that some teachers include in notion active teaching method also the methods of assessment during that are students doing some activities and that motivates students to work harder at their lessons.

Some teachers also use other active methods, such as the creation of mathematical bulletin boards, project learning, mathematical correspondence seminars, creating conceptual maps and similar activities.

Q3: Active methods are used mostly in the form of group teaching or frontal teaching and less in the form of individual teaching. That is in accordance with typical organization of educational process during teaching methods stated in Q2.

Q4, Q5: According the answer to the Q4 the majority of the teachers try to use active methods in all areas of mathematics. The numbers of positive responses to Q5 showed that teachers see difference in efficacy of using active methods between different areas of mathematics. The most respondents think that using active methods is the most effective in the area geometry and measurement. In this area of mathematics there are many very nice sources of active teaching methods [10], [11], [12], [13], [14]. In the following areas teachers use active methods with about the same efficacy: numbers, variables, computations with numbers; expressions, functions, tables, diagrams; combinatorics, probability and statistics. These areas are ideal for the use of ICT [15].

The lowest is using of active methods in the area logic, reasoning and proofs. This fact can encourage further study on the using of active methods in this area and also development of suitable teaching materials.

Some teachers think that the highest efficiency of active methods is in the areas in which they feel most certain themselves and have the deepest knowledge of them. Several teachers see as one of the most attractive topic financial mathematics, in this topic they have success using active methods effectively.

Q6, Q7: Respondents mostly use active methods in all levels of mathematics education. As for upper secondary education, respondents are using active methods less in the last year, which is associated with the preparation for graduation and entrance exams for universities those take part in this year of education. Distribution of responses is also related to the opinions and experience of some teachers that the effectiveness of active teaching methods depends not on the particular year of education, but on a particular group of students, respectively, some teachers think it does not depend neither on that.

Q8: Teachers use active methods in various phases of the lesson, the most in the motivational phase and none of them use them in the form of the homework.

Q9: When using active methods teachers focus mostly on these things: that the contexts of tasks are actual and based on reality; the development of pupils ability to communicate mathematics concepts and to use proper mathematics language; work with ICT and with information.

Q10: Most respondents during their teaching practice formed or are forming collection of activities those belong to active teaching methods. Most often teachers create these activities themselves, or they use online sources. Some teachers use the Internet in developing of their own activities only to update information, to obtain data on the history of mathematics; others search for complete activities ready for use. Many respondents draw inspiration or complete specific active methods from existing collections of tasks and activities. Some teachers use ideas or activities provided to them by their colleagues, those have successfully tested these activities in practice. Nowadays, teachers can took ideas also in the courses and trainings of further teacher education; this way used also some of our respondents.

Q11: Teachers provide us with a wide range of different activities and methods, even whole collections of their own activities. The most frequently have been mentioned didactic games and competitions. Often they were crossword puzzles, Sudoku and Bingo, but the most common activity was Algopretek, stated by about a quarter of respondents.

Conclusion

In this paper we presented survey on using of active teaching method in mathematics education at secondary schools. The aim of our survey was to determine if selected 20 mathematics teachers in Bratislava apply active teaching methods, which of them they use, in which conditions and their opinions related to this area.

As showed our survey, the majority of respondents are aware of the importance of the using of active methods in mathematics education and they include them systematically in the educational process. This result is in accordance with previous studies those have stated interest of teachers to innovate their methods in Slovakia [5]. A positive signal is also a diversity of methods and effort to find new inspirations, whether from the Internet, existing literature, or from colleagues and also creation of their own activities.

Because of the small study sample we cannot make any general conclusions but we got insight to the using of active teaching methods of selected mathematics teachers and also their best practices. This insight can contribute with some ideas to educators developing materials for active learning and also to other mathematics teachers in practice.

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